Improving Proof-Writing Skills
Through Weekly Student Presentations

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Abstract

Students acquire and hone proof-writing skills using an approach that features presenting proofs at the whiteboard in real analysis and in calculus. Preparing, presenting, and critiquing proofs engages students and effectively promotes their learning. The weekly schedule of instructor lectures and student presentations promotes a disciplined and efficient progression through the course content.

Difficulty Level: Medium; Course Level: Non-traditional, Advanced

1 Background and Context

I assign weekly student presentations to improve proof-writing skills at Bridgewater State University (BSU). BSU is a former teacher-preparation college now serving 11,000 students in southeastern Massachusetts. Approximately 300 students have mathematics as their declared major [2]. Most of them pursue careers in education.

I use the presentation technique to teach proof-writing to students in real analysis and calculus. In particular, I have used the approach for two semesters of Introduction to Real Analysis I, two sections of which were offered each semester. I have also used it in two sections of Calculus II, having students present proofs on the convergence or divergence of series. In the calculus class, the students not only identify which convergence test(s) to use, but also meticulously show how each hypothesis of the test is met. They state the conclusion in a complete sentence, citing the name of the test, for example, "Therefore, the series diverges by the limit comparison test." This work provides a basis for the formal structure students need in upper-level proof-based courses.

The first class in real analysis with student presentations had twelve students, who presented in pairs each week. The other section taught by another professor with other techniques had a similar number of students. The second class in real analysis that I taught with student presentations had eight students. Another professor taught a full-capacity section of 30 students using other techniques. All sections consisted of mathematics majors, primarily seniors, as well as a few juniors. Analysis students have learned proof techniques in a required freshman-level course, Transition to Advanced Mathematics. They have taken multivariable calculus and linear algebra as prerequisites and have had periodic exposure to proofs throughout their careers. However, at the beginning of the course, many of my students in real analysis reported that they did not feel confident in writing proofs.

In recent semesters, BSU has offered a few sections of Calculus II, generally capped at 25 students each. The two calculus classes in which I required student presentations had about 20 students each and consisted of approximately half mathematics majors and half science majors. Of the mathematics majors, most had taken Transition to Advanced Mathematics or were taking it concurrently with Calculus II. About half of the Calculus II students were sophomores, with the remaining students primarily first-year students for one section and primarily juniors for the other section.

All classes doing presentations had mathematics-education students well represented. About half of the students presenting proofs had double majors in elementary education or minors in secondary education.

Planning my course in real analysis, I believed that student presentations would engage my class deeply in learning to write proofs. I learned about implementing student presentations in a Project NExT session "Getting Started in Inquiry-Based Learning" at the Joint Mathematics Meetings in 2012. I based my system on that of Dr. Robert Vallin at Slippery Rock University, who requires student presentations in a course on advanced calculus [3]. Dr. Vallin had reported in correspondence with me that the vast majority of his students in real analysis were grateful for how much their abilities and confidence in writing proofs increased through giving presentations. I found the method so effective that I implemented it in Calculus II the next semester for teaching writing proofs on the convergence of series. I had taught such proofs over the previous ten years using traditional methods.

2 Description and Implementation

2.1 Overview

My presentation method requires devoting approximately 50 minutes per week to student presentations at the whiteboard. The time slot should occur at the same time every week to help the students and instructor get into a rhythm of preparing for class activities. For example, the presentations can take place during a 50-minute class meeting on a particular day of the week. For a class with a 75-minute meeting, the presentations can constitute the first 50 minutes of a class.

The rhythm of regular presentations promotes efficiency in lectures during the remaining class meetings. Following a careful weekly schedule promotes a prompt progression to new material and a comfortable pacing of the same course content that is covered without presentations. (See Section 2.5 on content for details.)

In my method of proof-writing instruction, the students work on homework prior to presentation of their proofs. I use different modes and timelines for initial homework in analysis and calculus. (See Section 2.3 on homework drafts for further details.)

In real analysis I collect a homework draft in the class period prior to presentations. Then I return graded homework drafts at the beginning of class on presentation day. Students receive grades for written homework drafts and presentations.

When students in Calculus II arrive in class on presentation day, they have been working on their online homework, which will come due at the start of the next class. Three tries per problem are available, and the online system provides feedback on whether each answer submitted is correct. So, students already know which of the online homework answers they have submitted so far are right and which are wrong when they present, and they may have additional attempts still available on each problem in their online homework. Not surprisingly, students who work on drafts before presentation day find presentations less stressful, and the vast majority of students do the advance work.

Before class on presentation day, I write approximately six problem numbers from the homework on the whiteboard, one problem per panel of the board, followed by the name(s) of the presenter(s), as well as, for a larger class, the name(s) of student(s) spearheading the class discussion of the problem. Explicitly flagging some students to ask questions promotes more discussion.

At the beginning of class on presentation day, the students look over their work on the homework and the feedback from me (Real Analysis) or online homework system (Calculus II) and compare notes with their partners if applicable. The presenters write their proofs on the board, finding a fair way to share the writing and speaking responsibilities. Sometimes one writes and the other speaks. Other times they split the writing and speaking. A student whose partner unexpectedly misses class must write and present solo. The absent student receives a score of zero. There are no make-up presentations.

After the presentation of a proof, the questioner(s) assigned to the problem must ask the presenter(s) a question or give a summary of the solution. They are graded on their participation. (See Section 2.4.) To promote honest and challenging discussions, student questions and critiques do not have a negative impact on the presenters' grades.

2.2 Partners

For large enough classes, I have students present with partners. The collaboration provides an additional layer of communication about proof-writing, which precedes the presentations and the subsequent discussion period. Working in pairs also helps each student feel less like a single individual put on the spot. Furthermore, teaming up with another student is a valuable networking opportunity, particularly on a campus like BSU, where most students are commuters.

In my experience, pairing responsible students together often results in respectful collaboration and complementary insights. I have seen less responsible students become somewhat more self-aware when working with similarly unreliable partners.

Switching the groupings week to week gives the students the opportunity to interact with a variety of collaborators. With changing partners, students can't rely on a particular skilled classmate every week. In larger classes with designated questioners, one should mix up the questioner and presenter roles from one presentation day to the next.

In my classes, the presentation method allowed for accommodation of and capitalization on a variety of cognitive styles. For example, two of my students who professed and manifested extreme anxiety over presentations took ownership of questioner roles; I never swapped them into the presenter role. They raised some of the best questions and consistently made helpful comments. Their forthright private disclosures of what was essentially a distracting disability led to enriched educational experiences, for the students directly affected and for their classmates.

I tell students that they can sell the skills they have used in class during job interviews. Employers value candidates' experience in preparation, collaboration, presentation, and critique.

2.3 Homework Drafts

Before presentations, students write an individual homework draft. In Real Analysis, it is submitted for a grade. In Calculus II, the students work on their online homework, writing a draft of the work in a homework notebook, which is not submitted for grading. After presentation day, students in both classes should polish their drafts in preparation for taking exams, but they do not submit a final written version of the homework for grading. In Calculus II, given that students can try each online problem three times, they may use any additional attempts on their online homework that are still available between presentation day and the due date.

Initially creating a draft rather than a finished product can set good priorities in the creative process of the writing of proofs: figure out the key ideas first; polish later. Students learn that the planning phase should precede the writing of a line-by-line proof. The time students invest in messy work on scratch paper improves their understanding of the problem, clarifying the task of designing a proof.

A draft in real analysis exposes starkly what the student knows and does not yet know. In calculus, a student can take an initial pass at developing an intuition for whether series converge or diverge before plunging into the proofs. The instructor can require students to write drafts in less time than the full week typically allotted for complete assignments, keeping the course moving along briskly.

2.3.1 Real Analysis

In Real Analysis, I grade a written draft, according to a simple rubric, and return the work in the next class. On a scale of a few points, maximum points correspond to having virtually all of the assignment completed, with answers clearly written to show excellent understanding. One point indicates a poor start on the homework or poor understanding. I write general comments on the drafts. The more detailed discussion takes place between partners and with the class during student presentations.

While grading drafts, I take notes about which students should present which problems. Sometimes a student with a particularly interesting solution should present it. Other times class presentations should bring to light a common error, which can then be discussed in a supportive environment in which I emphasize that the mistake was widespread and easy to make. Students who should work harder on a final solution than they did on the draft can present the problem and answer follow-up questions. One can assign a harder problem to a student who did a more routine presentation the previous week and vice versa. Students should email the professor if possible if they know they will miss presentation day to help with planning and organization and make the presentation session run smoothly.

In Real Analysis, the homework drafts and presentations each count for 15% of the grade in the course. Three exams count for 45% of the grade and a final exam for 25%. To reduce the pressure of the tight timeline for weekly homework drafts, I drop the lowest grade(s) for each student at the end of the semester. Both homework drafts and presentations are given significant weight to motivate the students to take them seriously and to do good work.

2.3.2 Calculus II

Prior to presenting proofs in Calculus II, students work on preliminary homework assignments online. They write accompanying work in a homework notebook but do not submit it for grading. The online system gives automatic feedback on their determination of convergence or divergence of a series and various aspects of the application of tests for convergence. The online homework comes due at the start of the class following presentation day.

In Calculus II, quizzes can help motivate students to do preliminary work before presentations. I give quizzes during the class prior to presentation day. The quizzes often cover the first couple of problems from the online homework and count for 15% of the course grade. Presentations count for ten percent. The online homework counts five percent. As in Real Analysis, three exams count for 45% of the grade and a final exam for 25%.

2.4 Student Presentations

To launch the presentations, I write approximately six problem numbers on the whiteboards around the room before class. Next to each problem, I write the names of the student(s) assigned to the problem. Depending on the material, one can assign two problems to one pair of students. Each student picks up his or her graded homework draft and discusses the presentation assignment with a partner if applicable, potentially even before class officially starts.

For Real Analysis, the drafts show me which problems are most beneficial to discuss. In Calculus II, students can anonymously submit requests for presentation problems during the preceding class.

Presentations begin when the students have completely written problem solutions on the board, usually ten minutes into the class period, and when everyone sits down. Students who don't quite finish writing sometimes do a bit of writing during their presentations. Presentation and subsequent discussion times vary according to the nature of the problem but can average about five minutes per presentation, with the whole process taking about 50 minutes of class time.

In the initial discussion between partners, students often find that they received a different written comment on the graded draft. Talking about the instructor feedback helps the team correct and improve their drafts, leading to consensus on how to write the proof on the board.

Occasionally students cannot write a proof and leave the board essentially blank. In this case, the problem receives minimal attention during presentations and I make encouraging comments about next time. In this situation, the professor has the option to provide or require some follow-up materials for distribution in the next class. The “failure” of a presentation can point the instructor to issues that befuddle the class as a whole, not just one pair of students.

Students receive a grade on a scale of five points with five corresponding to a fully correct presentation with good understanding. A student who requires tips or hints to give a correct presentation receives four points. A partially correct presentation earns three points. One showing weak understanding gets two points. Students who decline to present receive one point for attendance.

A similar five-point rubric applies to the work of questioners with five points for a question, comment, or summary that provides the class with additional insight, allows the class to clear up a misunderstanding, or reinforces or illuminates how to structure the proof, how to apply proof techniques, how to do the proof another way, or how to avoid errors. A student who is less able to raise a pertinent question or provide additional insight into the proof receives four points. Three points go to a questioner who is vague or shows weak understanding of the proof, two points for substantially vague input or a summary that shows substantially limited understanding of the proof. Students who decline to ask a question or to summarize receive one point for attendance.

One can post the presentation-day grades on the course-management website before the next class. An accompanying comment can come directly from the rubrics. Alternatively, one can write the presentation score on the next assignment the student submits. During presentations, I aim particular probing questions at each student, and partners receive independent grades, depending on their performance and understanding. I lower the stakes for presentations, putting the students more at ease, with a policy to drop each student’s lowest presentation grade(s) at the end of the semester.

2.5 Course Content

To compensate for time spent doing presentations, I distribute notes on the new material and present them at the document camera rather than writing them out in real time at the board. This strategy emphasizes the content over the act of writing. Examples in lectures are chosen judiciously, as many problems will receive detailed attention on presentation day.

A significant portion of presentation time comes from time that I would otherwise devote to miscellaneous questions during class throughout the week. Students must ask their homework questions during presentations, before class, after class, or in office hours.

One can also eliminate review days. Also, students continue to have homework in progress during exam weeks, but draft submission is optional. Presentations can take place on a volunteer basis or after more extensive discussion in pairs.

Using these strategies and keeping to a strict rhythm makes the course pacing more predictable and also more streamlined. In one semester I actually managed to cover two more sections in Calculus II than I had for all the previous sections I’d taught that didn’t do presentations, despite two days of campus closure due to snow storms. I covered approximately the same material as the other faculty, who taught the course without presentations.

3 Outcomes

3.1 Fewer Errors

This method seems to be effective in virtually eliminating my “favorite” error in proof-writing in each course. I have also seen a reduction in other common errors.

Before I implemented student presentations, my calculus students habitually abused the nth-term test for divergence. I had tried a variety of strategies to mitigate this error, such as having students meet with me for one-on-one discussions of the theorem, do targeted assignments, write up exam corrections, and recite the theorem like the pledge of allegiance. The sweat, tears, and humor did not work as well as I wanted. Now my students rarely if ever misapply nth-term test for divergence.

One reason is that a collection of my students' presentations on convergence and divergence proofs can serve to introduce and reinforce proof ideas from the BSU course on transition to advanced mathematics. The presentations provide strong vivid repetitive illustrations of how to apply a theorem: show the hypotheses are satisfied, state the conclusion, and cite the theorem by name. Also, the associated class conversations let us state the nth-term test for divergence over and over again in context, assess its applicability, experience its power, and reinforce an intuitive understanding of the theorem on a variety of problems.

When preparing for Real Analysis, I dreaded having to repeatedly correct standard errors of proof. For me a particular point of sensitivity is a student's assumption that p implies q while attempting to prove p implies q. Some professors in my department have had endemic cases of this error in upper-level courses with the same prerequisites. Fear of this error spurred me to try the student-presentation method.

My students in Real Analysis rarely make this error. In discussions during student presentations we explicitly emphasize the use of valid techniques, and we note avoidance of invalid techniques, referring repeatedly to a handout on the "Dos and Don'ts of Proof."

In both Calculus and Real Analysis, students learn from their peers' additional errors during presentations. They subsequently avoid them.

By way of example, in calculus a pair of presenters wrote very few words in applying the limit comparison test. The students had failed to justify several of the steps in their proof. Class discussion included a comparison between the test itself and how the students had applied it. Conveniently, another pair of students on the same day showed a very thorough application of the limit comparison test, and a comparison/contrast of their work with the less satisfactory proof drove home how to supply all the required reasoning when applying the test.

3.2 Greater Clarity

Students refine their proofs in presenting them. I watch this process in action when I see a student in real analysis read my feedback on a homework draft, rethink the problem, discuss the problem with a classmate, and present an improved proof. Students notice their own improvement, too. About a quarter of one class mentioned on exit surveys that explaining their work clarified their own understanding when asked "What did you like about the course?"

In calculus, preliminary work on online homework can give students the wrong idea that determining convergence or divergence is the extent of the task. My quizzes before presentation day often reveal inadequate proofs. The next day, I hold presenters to a high standard in the writing of a proof to support the convergence or divergence "answer," and students rise to the occasion, finding and appropriately using the tools that justify their conclusions. They improve upon their quiz proofs.

My students often appear to obtain a deeper understanding from their fellow students' correct answers to their questions than from my answers. They learn effectively from peers, incorporating the acquired knowledge into their proof-writing.

3.3 More Diverse Problem-Solving Strategies

Often for a problem, different students write different proofs and use different organizational strategies. Classmates gain a broad exposure to problem-solving strategies through a student presentation and associated discussion, which brings in ideas and comments on varied approaches to proof.

Presentations on calculus proofs lead to rich discussions of which other convergence tests are and are not applicable to a particular series. Students also discuss how they got an intuition for the behavior of the series as a springboard to selecting an applicable convergence theorem and constructing a proof.

Students in real analysis similarly engage with the question of how to design a bridge from hypotheses to conclusion. For example, one student started her presentation by saying, "At first I thought my partner's answer was wrong and mine was right because I got full credit on my draft. But she thought I was wrong because she got full credit. Then we realized we were both right." Then the students presented the two ways and explained their compatibility.

When classmates write proofs after such presentation sessions, they bring more experience, tools, and perspectives to bear. Sometimes they discuss and compare them with me in office hours. Then they write a strong proof. In addition, discussion of a variety of student approaches lets students overcome a fear of not knowing "the" right way. Such anxiety can be a barrier to seeking an effective proof strategy.

3.4 Improved Attitudes

As it happens, consolidating homework discussion to presentation day serves my students well. Most arrive in class ready to meaningfully discuss the problems. Prior to my implementation of the method, some students noted on course evaluations that they found in-class Q&A too individually targeted and too repetitive. At the time, most students had not benefited from those teachable moments on proof-writing because they had not worked on the problem and did not yet feel invested in it.

As an added benefit, the presentation method appears to increase my students' motivation. For example, unlike students in the other sections of real analysis, almost all my students submit their drafts religiously even though they know I will drop the lowest three grades at the end of the semester.

In my classes, the presentation method has dispelled the culture of "I can't do proofs" or "I don't get series." The class environment makes these assertions socially unacceptable. Doing proofs becomes part of the expected routine. Students must and do spend significant time on the task. Ultimately, the presentation/discussion period provides an outlet for the enthusiasm of the instructor and the students. In both real analysis and calculus, the presentations produce an enjoyable atmosphere in the classroom.

4 Extending the Method

The description of the method in Section 2 indicates that the number of presenters on each of about six problems depends on the size of the class. In my calculus classes of about 20, I assign two presenters to each problem and also designate a student to lead class discussion of the problem with a question or a summary. In larger classes, one can assign additional students to ask questions about every problem presented.

This method works best with at least six panels of board space available for putting up a problem on each panel. A room with only four panels of board space can also accommodate the technique if a couple of presentations can be done via a document camera. Alternatively, the number of proofs presented can shrink from about six to four if the room has four panels and no document camera.

If the size of the class permits, all students should have roles presenting or questioning on each presentation day. Otherwise, larger classes can have each student participating in presentations every other week instead of every week. (Editors' note: For another approach to student presentations, see Mathews's paper [1] in this volume.)

Also, the presentations can be embedded in an inquiry-based course. For example, Dr. Vallin teaches Advanced Calculus With Generalizations in this form, covering similar topics in the first semester to the course Introduction to Real Analysis I, addressed here. He has published a full set of notes [3].

References

[1] Bryant G. Mathews, "Individual Presentations of Group-Written Proofs," (in this volume).

[2] Office of Institutional Research, Bridgewater State University, "Bridgewater State University Factbook 2013–2014", Bridgewater State University, at `http://www.bridgew.edu/the-university/bsu-facts`, accessed December 2014.

[3] Robert W. Vallin, "Advanced Calculus with Generalizations: First Semester," *Journal of Inquiry-Based Learning in Mathematics*, vol. 34, 2013.

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