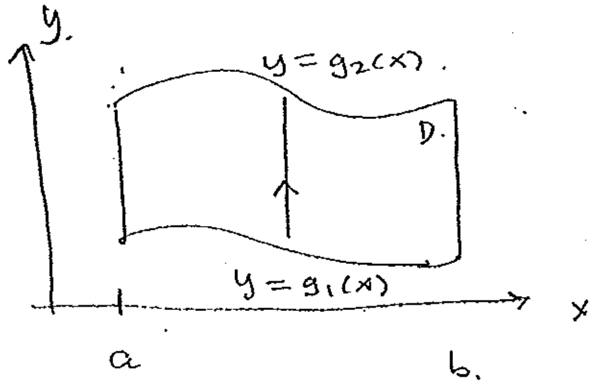
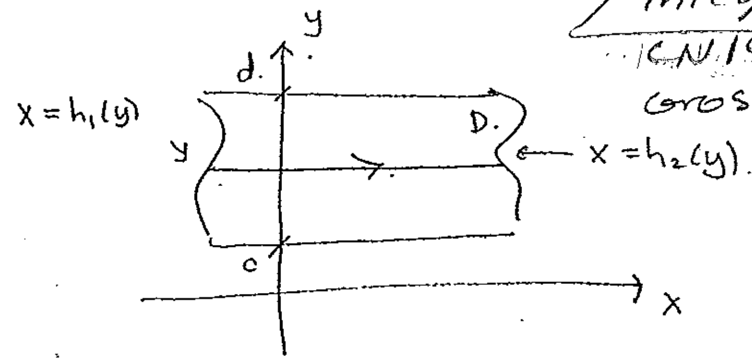
① D could be "type I" ② or "type II." Area integrals CN 19 Gross.





$$\iint_D f(x,y)\,dA = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy \right] dx$$

$$\iint_D f(x,y)\,dA = \int_c^d \int_{x=h_1(y)}^{x=h_2(y)} f(x,y)\,dx\,dy$$

③ D could be a "polar region"

Recall we had in Calc III (multivariable calc).

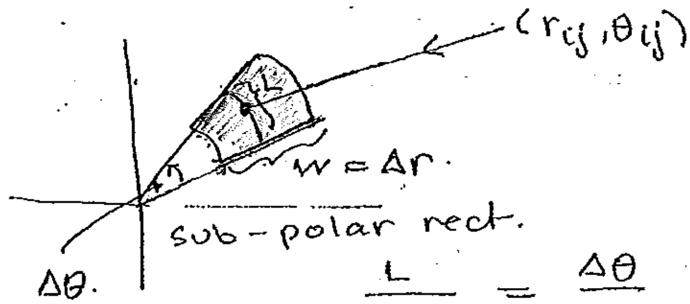
$$\iint_D f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}, y_{ij})\,\Delta A.$$

$\Delta A = ?$

$\Delta A \approx w \cdot L$

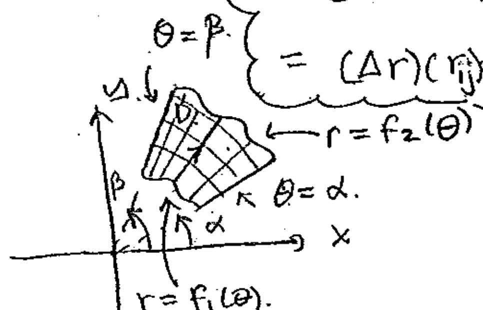
$= (\Delta r)(L)$

$= (\Delta r)(r_{ij})(\Delta\theta)$



$$\frac{L}{2\pi r_{ij}} = \frac{\Delta\theta}{2\pi}$$

$$L = r_{ij}\,\Delta\theta.$$



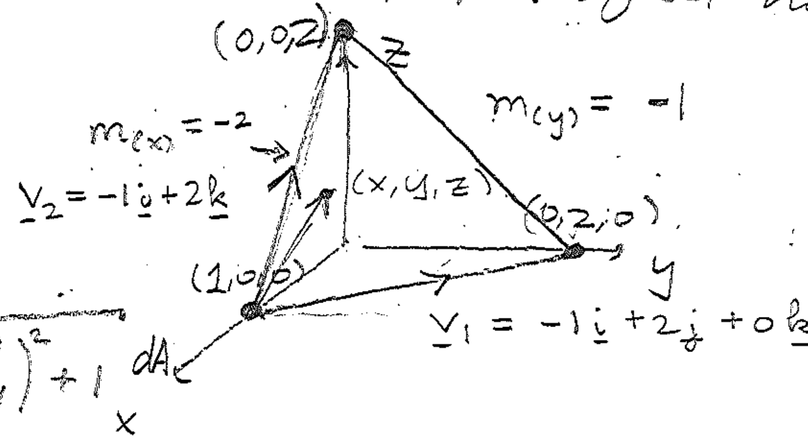
$$= \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(r_{ij}\cos\theta_{ij}, r_{ij}\sin\theta_{ij})\, r_{ij}\,\Delta r\,\Delta\theta.$$

$$= \int_\alpha^\beta \int_{f_1(\theta)}^{f_2(\theta)} \underbrace{f(r\cos\theta, r\sin\theta)}_{\text{e.g. } \frac{\text{mass}}{\text{area}}}\, \underbrace{r}_{\text{dist}}\, \underbrace{dr}_{\text{dist}}\, d\theta$$

units of mass

e. 1. Find the mass of a flat triangular sheet with vertices $(1,0,0)$, $(0,2,0)$, & $(0,0,2)$ if it has density $\rho(x,y,z) = xy$.

area unit, z

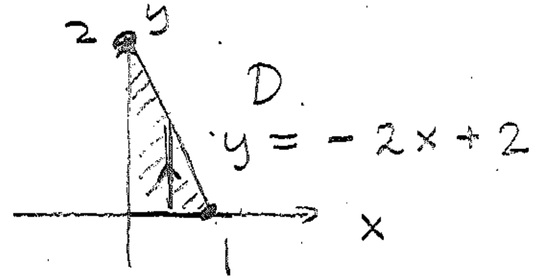


$$M = \iint_S x\,y\, dS \qquad S \text{ is } z = g(x,y);$$

$$= \iint_D \rho(x,y,g(x,y)) \sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2 + 1}\; dA$$

$$\frac{\partial z}{\partial x} = -2, \quad \frac{\partial z}{\partial y} = -1$$

$$= \iint_D x\,y \sqrt{(-2)^2 + (-1)^2 + 1}\; dA$$



$$= \int_0^1 \int_0^{-2x+2} x\,y \sqrt{6}\; dy\,dx$$

$$-2x+2 = 2(-x+1)$$

$$= \sqrt{6} \int_0^1 \frac{x\,y^2}{2}\Bigg|_{y=0}^{} dx = \frac{\sqrt{6}}{2} \int_0^1 x\,[2(-x+1)]^2\, dx = \frac{\sqrt{6}}{\cancel{2}} \int_0^1 x\,\cancel{4}^{\,2}(x^2 - 2x + 1)\,dx$$

$$= 2\sqrt{6} \int_0^1 [x^3 - 2x^2 + x]\, dx = 2\sqrt{6}\left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right]\Bigg|_0^1$$

$$= 2\sqrt{6}\left[\frac{1}{4} - \frac{2}{3} + \frac{1}{2}\right] = 2\sqrt{6}\,\frac{3 - 8 + 6}{12} = \frac{2\sqrt{6}}{12} = \frac{\sqrt{6}}{6} \text{ units of mass}$$