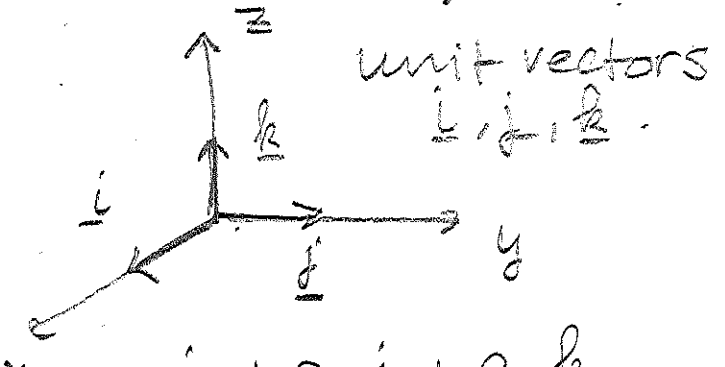
CW17: Cross product or vector product

$\underline{a} = (a_1, a_2, a_3)$, $\underline{b} = (b_1, b_2, b_3)$

$\underline{a} \times \underline{b} = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1)$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$



unit vectors $\underline{i}, \underline{j}, \underline{k}$.

$a_1 \underline{i} + a_2 \underline{j} + a_3 \underline{k}$
$= (a_1, a_2, a_3)$

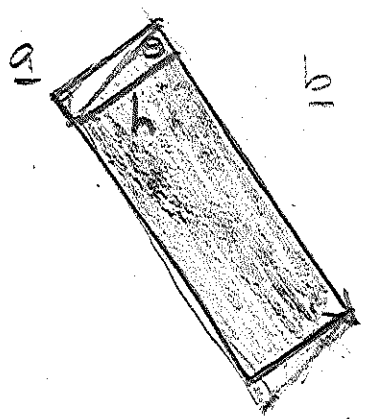
$$= \underline{i} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \underline{j} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \underline{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= \underline{i}(a_2 b_3 - a_3 b_2) - \underline{j}(a_1 b_3 - a_3 b_1) + \underline{k}(a_1 b_2 - a_2 b_1)$$

$$= (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1) \checkmark$$

$\|\underline{a} \times \underline{b}\| = \|\underline{a}\| \|\underline{b}\| \sin\theta$

$\|\underline{a} \times \underline{b}\|$ = area of parallelogram determined by $\underline{a}$ & $\underline{b}$.



area of parallelogram = area of rectangle $= \|\underline{b}\| h = \|\underline{b}\| \|\underline{a}\| \sin\theta$

$\sin\theta = \frac{h}{\|\underline{a}\|} \rightarrow h = \|\underline{a}\| \sin\theta$

$\underline{a} \neq \underline{0}$, $\underline{b} \neq \underline{0}$: Vectors $\underline{a}$ & $\underline{b}$ are orthogonal if and only if $\underline{a} \cdot \underline{b} = 0$

$\underline{a} \neq \underline{0}$, $\underline{b} \neq \underline{0}$: Vectors $\underline{a}$ & $\underline{b}$ are parallel if and only if $\underline{a} \times \underline{b} = \underline{0}$

what direction is $\underline{a} \times \underline{b}$? $\underline{a} \times \underline{b} \perp \underline{a}$ & $\underline{a} \times \underline{b} \perp \underline{b}$.